Supplementary Material for "A Survey of Evolutionary Algorithms for Multi-Objective Optimization Problems With Irregular Pareto Fronts"

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I. MOEAs for MOPs With Irregular PFs

In this section, we are going to provide an overview of existing MOEAs dedicated to solving MOPs with irregular PFs. Here, the MOPs are categorized into four groups according to their main mechanism for handling irregularity in the PF.

A. Fixed Vector Based MOEAs

The fixed vector based MOEAs for MOPs with irregular PFs are summarized in Table SI.

B. Reference Vector Adjustment Based MOEAs

The reference vector adjustment based MOEAs for MOPs with irregular PFs are summarized in Table SII.

C. Reference Point Based MOEAs

The reference point based MOEAs for MOPs with irregular PFs are summarized in Table SIII.

D. Grid or Clustering Based MOEAs

The grid or clustering based MOEAs for MOPs with irregular PFs are summarized in Table SIV.

TABLE SI
Summary of Fixed Vector Based MOEAs for Irregular PFs

MOEA	Auxiliary methods
BCE-MOEA/D [1]	Two populations evolve based on non-dominated sorting and decomposition in parallel
MOEA/D-AED [2]	Non-dominated individuals with small Chebyshev function value are stored in an external archive
MOEA/D-SAS [3]	Select more than one individual by one reference vector. The individuals in each sub-problem are sorted first according to their fitness value, and then according to the angles between other individuals
ASEA [4]	Select more than one individual by one reference vector. The individuals in each sub-problem are first sorted by convergence, and then by an angle based crowding degree evaluation method
PAEA [5]	Using two fixed reference vector sets using the ideal point and the nadir point as the origin respectively. PBI and inverted PBI scalarizing functions are simultaneously used
MOEA/AD [6]	Using two fixed reference vector sets using the ideal point and the nadir point as the origin respectively. PBI and an augmented achievement scalarizing function (AASF) are adopted for each set of the reference vectors
MOEA/D-MR [7]	Using two fixed reference vector sets using the ideal point and the nadir point as the origin, respectively. An archive is used to store non-dominated individuals
MOEA/D-TPN [8]	Two fixed reference vector sets using the ideal point and the nadir point as the origin respectively are used in two stages
DBEA-DS [9]	Two sets of solutions are obtained in each generation with two sets of reference directions. Only one set of solution of the least fitness value will survive.

TABLE SII
Summary of Reference Vector Adjustment Based MOEAs for Irregular PFs

MOEA	Reference vector adjustment	Adjustment criterion
	Utilizing solutions to generate reference vectors	
MOEA/D-AWA [10]	Crowded reference vectors will be deleted and new reference vectors will be added in sparse regions using the solutions in the archive	Every several generations after a number of fixed generations
FV-MOEA/D [11]	Use the solutions in an archive to generate new reference vectors	Every several generations
AdaW [12]	A set of solutions are picked up from the archive to generate the corresponding weight vectors,delete poor weight vectors	Every several generations
MOEA/D-URAW [13]	Add weight vectors to sparse areas and delete weight vectors in crowded areas	Every generation after a number of fixed generations
E-IM-MOEA [14]	Generation the reference vectors using the solutions in the archive	Every generation at the late stage of the search process
EARPEA [15]	New reference vectors generated by selecting solutions from the non-dominated solutions from the current population according to the cosine distance	Every generation if the number of active reference vectors is insufficient

Table SII (Continued)

MOEA	Reference vector adjustment	Adjustment criterion
[16]	Use solutions both in the archive and in the current population as the candidate reference vectors	Every generation after a number of fixed
VaEA [17]	Individuals with larger angle with the selected individual vectors are selected as vectors	Every generation
LC-MaOEA [18]	The cluster center vectors generated by clustering the individuals mapped on the hyperplane are used as the reference vectors	Only once after the active
MOEA/D-AM2M [19]	Reference vectors are generated by picking up a solution vector with the biggest angle to the existing reference vectors	Every several generations after a number of fixed generations
EMOSA [20]	Ajusting weight vectors to make each individual away from its nearest neighbor	Every several generations if the lowest temperature is reached
APA [21]	The solutions contributing more to the hypervolume of the current population will be picked up to generate the reference vectors	Every generation
g-DBEA [22]	An inactive reference vector is replaced by the solution with the maximum angle to its neighbouring reference vectors with more than one solution attached to	Every generation
iRVEA [23]	Selecting solution vectors having the largest cosine distance to the existing active reference vectors to replace the inactive reference vectors	Every generation
	Adjusting reference vectors using existing ones	
TPEA-PBA [24]	New reference vectors are generated by disturbing the "good" reference vectors based on the penalty values	Only once in final 50 generations
MaOEA/D-2ADV [25]	Inactive reference vectors are replaced by the interpolation between active reference vectors based on the Euclidean distance between the reference vectors	Every several generations
MOEA/D-ABD [26]	The weight vectors are divided into two sets according to whether they have intersections with the PFs and are adjusted using different step sizes accordingly within each discontinuous segment of PFs	Every several generations after the endpoints of each discontinuous region of the PFs are detected
A-NSGA-III [27]	Reference vectors are sampled around the promising reference vectors whose niche count is more than one	Every generation
A2-NSGA-III [28]	Reference vectors are sampled around the promising reference vectors whose niche count is more than one	Every generation
AMOEA/D [29]	Remove invalid reference points and add several points around crowded reference point	Every several generations after indicator MDP is satisfied
NSGA-MPBI [30]	Denser reference vectors are generated until the number of active reference vectors is larger than the population size	Every several generations
	Learning the distribution of PFs	
MOEA/D-SOM [31], M2M-SOM	Use the self-organising map (SOM) network to learn the topology of PFs, the nodes of the SOM serve as the positions of the reference vectors	Every generation
DEA-GNG [32]	Reference vectors are adjusted according to the topology of the PFs obtained by training a growing neural gas (GNG) network	Every generation
RVEA-iGNG [33]	An improved GNG (iGNG) is designed for adapting the reference vectors	Every generation
CLIA [34]	Delete inactive reference vectors detected by incremental support vector machine method, add vectors around active vectors	Every generation
Adaptive MOEA/D [35]	Reference vectors in different subregions are assigned based on the complexity of each region	Every several generations
MOEA/D-AWG [36]	Reference vectors are uniformly sampled on PFs learned by Gaussian process regression models	Every several generations after a number of fixed generations
MOEA/D-LTD [37]	Reference vectors are uniformly sampled on PFs learned by Gaussian process regression models	Every several generations after a number of fixed generations
PICEA-w [38]	The solutions in the population are coevolved with the reference vectors	Every generation

TABLE SIII
SUMMARY OF REFERENCE POINT BASED MOEAS FOR IRREGULAR PFS

MOEA	Reference point adjustment	Adjustment criterion
	Adjusting reference points using existing ones	
AR-MOEA [39]	Add the individuals with the largest angle from active reference points as new reference points, and delete inactive reference points	Every generation
AREA [40]	Add the projection of the individuals in the archive furthest from the current population as a new reference point, and remove the reference points with the worst index	Every several generations
	Utilizing individuals to generate reference points	
RPEA [41]	Individuals with the largest crowding distances are chosen to generate reference points by reducing the corresponding objective values	Every several generations
CA-MOEA [42]	Clustering individuals and calculating the cluster centers as reference points	Every generation

TABLE SIV
GRID OR CLUSTERING BASED MOEAS FOR IRREGULAR PFS

MOEA	Partition method	Partition criterion
	Grid based division	
RdEA [43]	Using the middle point between two endpoints to divide the sub region until the number of sub regions meets the requirements	Every generation
CDG-MOEA [44]	Segment the objective axis to create grids	Every generation
MOEA-PPF [45]	Partition the population from the discontinuous points detected by the crowding distance	Only once at half of the generations
	Clustering based division	
CLUMOEA [46]	K-means clustering	Every several generations
F-DEA [47]	Treat the active reference points as cluster centers to do the clustering	Every generation
EMyO/C [48]	Clustering individuals on the hyperplane	Every generation
MaOEA/C [49]	Firstly, the axises are used as the clustering centers for clustering. Then, in each cluster, hierarchical clustering method is used to cluster the non-dominated individuals	Clustering by axises only in the first generation, and Hierarchical clustering in every generation

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