Appendix for "Bipartite Formation Tracking for Multi-Agent Systems Using Fully Distributed Dynamic Edge-Event-Triggered Protocol"

Appendix
Proof of Theorem 1

Proof: Denote $\check{\xi}_i = \bar{x}_i^1 - H_i y_i - \check{x}_i^1$, $i \in \mathcal{V_F}$. Combining (4a) with (5a) gives

$$\dot{\check{\xi}}_i = (\check{A}_i^{11} + H_i \check{A}_i^{21})\check{\xi}_i. \tag{12}$$

Since $\check{A}_i^{11} + H_i \check{A}_i^{21}$ is Hurwitz, one has $\lim_{t\to\infty}(\bar{x}_i^1 - H_i y_i) = \check{x}_i^1$. It is further obtained that $\lim_{t\to\infty} \hat{x}_i = x_i$. Similarly, we have $\lim_{t\to\infty}(\bar{\zeta}_\epsilon^1 - H y_\epsilon) = \check{x}_\epsilon^1$ and $\lim_{t\to\infty} \zeta_\epsilon = x_\epsilon$. By noting that (8), it is easy to find that $(\bar{\zeta}_\epsilon^1 - H y_\epsilon - \check{x}_\epsilon^1)$ is decoupled from ξ_i. Therefore, the global convergence of (8) is equivalent to the global convergence of the following system:

$$\dot{\xi}_i = \tilde{A}\xi_i - \Gamma\Big(\sum_{j=1}^{M} \varsigma_{ij} |w_{ij}^{\varrho(t)}| \hat{y}_{ij} + \varsigma_i b_i^{\varrho(t)} \sum_{\epsilon=M+1}^{M+N} \hat{y}_{i\epsilon}\Big). \tag{13}$$

The Lyapunov function candidate is constructed as

$$V = W + \frac{\delta}{2}\sum_{i=1}^{M}\sum_{j=1}^{M} \Upsilon_{ij} + \frac{\delta}{2}\sum_{i=1}^{M}\sum_{\epsilon=M+1}^{M+N} \Upsilon_{i\epsilon} \tag{14}$$

where $W = (1/2)\sum_{i=1}^{M} \xi_i^T P \xi_i + \sum_{i=1}^{M}\sum_{j=1}^{M} (\varsigma_{ij} - \delta)^2/(8\kappa_{ij}) + \sum_{i=1}^{N} (\varsigma_i - \delta)^2/(4\kappa_i)$ with $\delta = \max_{i,j\in\mathcal{V_F}}\{4/\bar{\lambda}_{\min}, 1/\rho_{ij}, 1/\rho_i\}$ and $\bar{\lambda}_{\min}$ being the minimum nonzero eigenvalue of all possible $\mathcal{L}_1^{\varrho(t)}$, $\varrho(t) \in \Xi$. According to (6a) and (7a), it is easy to obtain that

$$\dot{\Upsilon}_{ij} \geq -(\tilde{\sigma}_{ij}\nu_{ij} + \bar{\sigma}_{ij})\Upsilon_{ij}. \tag{15}$$

Using the comparison lemma gives

$$\Upsilon_{ij} \geq e^{-(\tilde{\sigma}_{ij}\nu_{ij} + \bar{\sigma}_{ij})t}\Upsilon_{ij}(0) > 0. \tag{16}$$

Similarly, one has $\Upsilon_{i\epsilon} > 0$. Obviously, V is positive definite. When $\varrho(t) = s$, combining (13) with (14) gives

$$\begin{aligned}\dot{W} = &\sum_{i=1}^{M} \xi_i^T P\tilde{A}\xi_i - \sum_{i=1}^{M} \xi_i^T P\Gamma\Big(\sum_{j=1}^{M} \varsigma_{ij}|w_{ij}^s|\hat{y}_{ij} + \varsigma_i b_i^s \sum_{\epsilon=M+1}^{M+N} \hat{y}_{i\epsilon}\Big) \\ &+ \sum_{i=1}^{M}\sum_{j=1}^{M} \frac{\varsigma_{ij} - \delta}{4\kappa_{ij}}\dot{\varsigma}_{ij} + \sum_{i=1}^{M} \frac{\varsigma_i - \delta}{2\kappa_i}\dot{\varsigma}_i.\end{aligned} \tag{17}$$

Denote $\hat{e}_{ij} = e_{ij} - \mathrm{sgn}(w_{ij})e_{ji}$ and $\hat{e}_{i\epsilon} = e_{i\epsilon} - d_i e_\epsilon$. In light to the Young's inequality, we have

$$\begin{aligned}&-\sum_{i=1}^{M} \xi_i^T P\Gamma \sum_{j=1}^{M} \varsigma_{ij}|w_{ij}^s|\hat{y}_{ij} \\ &= -\frac{1}{2}\sum_{i=1}^{M}\sum_{j=1}^{M} \varsigma_{ij}|w_{ij}^s|(\hat{y}_{ij} - \hat{e}_{ij})^T\hat{y}_{ij} \\ &\leq -\frac{1}{4}\sum_{i=1}^{M}\sum_{j=1}^{M} \varsigma_{ij}|w_{ij}^s|\|\hat{y}_{ij}\|^2 + \frac{1}{4}\sum_{i=1}^{M}\sum_{j=1}^{M} \varsigma_{ij}|w_{ij}^s|\|\hat{e}_{ij}\|^2 \\ &\leq -\frac{1}{4}\sum_{i=1}^{M}\sum_{j=1}^{M} \varsigma_{ij}|w_{ij}^s|\|\hat{y}_{ij}\|^2 + \sum_{i=1}^{M}\sum_{j=1}^{M} \varsigma_{ij}|w_{ij}^s|\|e_{ij}\|^2\end{aligned} \tag{18}$$

and

$$\begin{aligned}&-\sum_{i=1}^{M} \xi_i^T P\Gamma\varsigma_i b_i^s \sum_{\epsilon=M+1}^{M+N} \hat{y}_{i\epsilon} \\ &= -\frac{1}{N}\sum_{i=1}^{M} \varsigma_i b_i^s \Big(\sum_{\tilde{\epsilon}=M+1}^{M+N} (\hat{y}_{i\tilde{\epsilon}} - \hat{e}_{i\tilde{\epsilon}})\Big)^T \sum_{\epsilon=M+1}^{M+N} \hat{y}_{i\epsilon} \\ &\leq -\frac{1}{2N}\sum_{i=1}^{M} \varsigma_i b_i^s \Big\|\sum_{\epsilon=M+1}^{M+N} \hat{y}_{i\epsilon}\Big\|^2 + \frac{1}{2N}\sum_{i=1}^{M} \varsigma_i b_i^s \Big\|\sum_{\epsilon=M+1}^{M+N} \hat{e}_{i\epsilon}\Big\|^2 \\ &\leq -\frac{1}{2N}\sum_{i=1}^{M} \varsigma_i b_i^s \Big\|\sum_{\epsilon=M+1}^{M+N} \hat{y}_{i\epsilon}\Big\|^2 + \sum_{i=1}^{M}\sum_{\epsilon=M+1}^{M+N} \varsigma_i b_i^s \|e_{i\epsilon}\|^2 \\ &\quad + \sum_{i=1}^{M}\sum_{\epsilon=M+1}^{M+N} \varsigma_i b_i^s \|e_\epsilon\|^2\end{aligned} \tag{19}$$

where we have used the facts that $\tilde{C}\xi_i - \mathrm{sgn}(w_{ij})\tilde{C}\xi_j = \hat{y}_{ij} - \hat{e}_{ij}$, $P\Gamma = \tilde{C}^T$, $\varsigma_{ij} = \varsigma_{ji}$ and $\tilde{C}\xi_i = (1/N)\sum_{\epsilon=M+1}^{M+N}(\hat{y}_{i\epsilon} - \hat{e}_{i\epsilon})$. It follows from (5d) and (5e) that:

$$\begin{aligned}&\sum_{i=1}^{M}\sum_{j=1}^{M} \frac{\varsigma_{ij} - \delta}{4\kappa_{ij}}\dot{\varsigma}_{ij} \\ &= \sum_{i=1}^{M}\sum_{j=1}^{M} \frac{\varsigma_{ij} - \delta}{4}|w_{ij}^s|\|\hat{y}_{ij}\|^2 - \sum_{i=1}^{M}\sum_{j=1}^{M} \frac{\varsigma_{ij} - \delta}{4}\iota_{ij}\varsigma_{ij} \\ &\leq \sum_{i=1}^{M}\sum_{j=1}^{M} \frac{\varsigma_{ij} - \delta}{4}|w_{ij}^s|\|\hat{y}_{ij}\|^2 - \sum_{i=1}^{M}\sum_{j=1}^{M} \frac{\iota_{ij}}{8}(\varsigma_{ij} - \delta)^2 \\ &\quad + \sum_{i=1}^{M}\sum_{j=1}^{M} \frac{\iota_{ij}}{8}\delta^2\end{aligned} \tag{20}$$

and

$$\sum_{i=1}^{M}\frac{\varsigma_i-\delta}{2\kappa_i}\dot{\varsigma}_i$$

$$=\sum_{i=1}^{M}\frac{\varsigma_i-\delta}{2N}b_i^s\|\sum_{\epsilon=M+1}^{M+N}\hat{y}_{i\epsilon}\|^2-\sum_{i=1}^{M}\frac{\varsigma_i-\delta}{2}\iota_i\varsigma_i$$

$$\le\sum_{i=1}^{M}\frac{\varsigma_i-\delta}{2N}b_i^s\|\sum_{\epsilon=M+1}^{M+N}\hat{y}_{i\epsilon}\|^2-\sum_{i=1}^{M}\frac{\iota_i}{4}(\varsigma_i-\delta)^2+\sum_{i=1}^{M}\frac{\iota_i}{4}\delta^2. \tag{21}$$

Note that

$$-\sum_{i=1}^{M}\sum_{j=1}^{M}|w_{ij}^s|\|\hat{y}_{ij}\|^2$$

$$=-\sum_{i=1}^{M}\sum_{j=1}^{M}|w_{ij}^s|\|\tilde{C}\xi_i-\mathrm{sgn}(w_{ij})\tilde{C}\xi_j\|^2-\sum_{i=1}^{M}\sum_{j=1}^{M}|w_{ij}^s|\|\hat{e}_{ij}\|^2$$

$$-2\sum_{i=1}^{M}\sum_{j=1}^{M}|w_{ij}^s|(\tilde{C}\xi_i-\mathrm{sgn}(w_{ij})\tilde{C}\xi_j)^T\hat{e}_{ij}$$

$$\le-\frac{1}{2}\sum_{i=1}^{M}\sum_{j=1}^{M}|w_{ij}^s|\|\tilde{C}\xi_i-\mathrm{sgn}(w_{ij})\tilde{C}\xi_j\|^2$$

$$+4\sum_{i=1}^{M}\sum_{j=1}^{M}|w_{ij}^s|\|e_{ij}\|^2 \tag{22}$$

$$-\sum_{i=1}^{M}b_i^s\|\sum_{\epsilon=M+1}^{M+N}\hat{y}_{i\epsilon}\|^2$$

$$=-N^2\sum_{i=1}^{M}b_i^s\|\tilde{C}\xi_i\|^2-\sum_{i=1}^{M}b_i^s\|\sum_{\epsilon=M+1}^{M+N}\hat{e}_{i\epsilon}\|^2$$

$$-2N\sum_{i=1}^{M}b_i^s(\tilde{C}\xi_i)^T\sum_{\epsilon=M+1}^{M+N}\hat{e}_{i\epsilon}$$

$$\le-\frac{N^2}{2}\sum_{i=1}^{M}b_i^s\|\tilde{C}\xi_i\|^2+2N\sum_{i=1}^{M}\sum_{\epsilon=M+1}^{M+N}b_i^s\|e_{i\epsilon}\|^2$$

$$+2N\sum_{i=1}^{M}\sum_{\epsilon=M+1}^{M+N}b_i^s\|e_\epsilon\|^2 \tag{23}$$

and

$$\frac{1}{2}\sum_{i=1}^{M}\sum_{j=1}^{M}|w_{ij}^s|\|\tilde{C}\xi_i-\mathrm{sgn}(w_{ij})\tilde{C}\xi_j\|^2+N\sum_{i=1}^{M}b_i^s\|\tilde{C}\xi_i\|^2$$

$$=\xi^T(\mathcal{L}_1^s\otimes\tilde{C}^T\tilde{C})\xi. \tag{24}$$

Substituting (18)–(24) into (17) yields

$$\dot{W}\le\frac{1}{2}\xi^T(I_M\otimes(P\tilde{A}+\tilde{A}^TP)-\frac{\delta}{4}\mathcal{L}_1^s\otimes\tilde{C}^T\tilde{C})\xi$$

$$+\frac{\delta}{2}\sum_{i=1}^{M}\sum_{j=1}^{M}\Big((1+\frac{2}{\delta}\varsigma_{ij})|w_{ij}^s|\|e_{ij}\|^2-\frac{1}{4}|w_{ij}^s|\|\hat{y}_{ij}\|^2\Big)$$

$$+\frac{\delta}{2}\sum_{i=1}^{M}\sum_{\epsilon=M+1}^{M+N}\Big((1+\frac{2}{\delta}\varsigma_i)b_i^s\|e_{i\epsilon}\|^2-\frac{1}{2}b_i^s\|\hat{y}_{i\epsilon}\|^2\Big)$$

$$+\frac{\delta}{2}\sum_{i=1}^{M}\sum_{\epsilon=M+1}^{M+N}(1+\frac{2}{\delta}\varsigma_i)b_i^s\|e_\epsilon\|^2$$

$$-\sum_{i=1}^{M}\sum_{j=1}^{M}\frac{\iota_{ij}}{8}(\varsigma_{ij}-\delta)^2-\sum_{i=1}^{M}\frac{\iota_i}{4}(\varsigma_i-\delta)^2+\pi \tag{25}$$

where $\pi=\sum_{i=1}^{M}\sum_{j=1}^{M}(\iota_{ij}/8)\delta^2+\sum_{i=1}^{M}(\iota_i/4)\delta^2$.

By noting that the triggering functions (7) and $\delta\ge\max_{i,j\in\mathcal{V}_\mathcal{F}}\{1/\rho_{ij},1/\rho_i\}$, combining (14) with (25) gives

$$\dot{V}\le\frac{1}{2}\xi^T(I_M\otimes(P\tilde{A}+\tilde{A}^TP)-\frac{\delta}{4}\mathcal{L}_1^s\otimes\tilde{C}^T\tilde{C})\xi$$

$$+\frac{\delta}{2}\sum_{i=1}^{M}\sum_{j=1}^{M}(1-\tilde{\sigma}_{ij})\Big((1+2\rho_{ij}\varsigma_{ij})|w_{ij}^s|\|e_{ij}\|^2$$

$$-\frac{1}{4}|w_{ij}^s|\|\hat{y}_{ij}\|^2\Big)$$

$$+\frac{\delta}{2}\sum_{i=1}^{M}\sum_{\epsilon=M+1}^{M+N}(1-\tilde{\sigma}_{i\epsilon})\Big((1+2\rho_i\varsigma_i)b_i^s\|e_{i\epsilon}\|^2$$

$$-\frac{1}{2}b_i^s\|\hat{y}_{i\epsilon}\|^2\Big)$$

$$+\frac{\delta}{2}\sum_{i=1}^{M}\sum_{\epsilon=M+1}^{M+N}(1+2\rho_i\varsigma_i)b_i^s\|e_\epsilon\|^2$$

$$+\frac{\delta}{2}\sum_{i=1}^{M}\sum_{j=1}^{M}\tilde{\sigma}_{ij}\tilde{\mu}_{ij}e^{-\bar{\mu}_{ij}t}-\frac{\delta}{2}\sum_{i=1}^{M}\sum_{j=1}^{M}\bar{\sigma}_{ij}\Upsilon_{ij}$$

$$+\frac{\delta}{2}\sum_{i=1}^{M}\sum_{\epsilon=M+1}^{M+N}\tilde{\sigma}_{i\epsilon}\tilde{\mu}_{i\epsilon}e^{-\bar{\mu}_{i\epsilon}t}-\frac{\delta}{2}\sum_{i=1}^{M}\sum_{\epsilon=M+1}^{M+N}\bar{\sigma}_{i\epsilon}\Upsilon_{i\epsilon}$$

$$-\sum_{i=1}^{M}\sum_{j=1}^{M}\frac{\iota_{ij}}{8}(\varsigma_{ij}-\delta)^2-\sum_{i=1}^{M}\frac{\iota_i}{4}(\varsigma_i-\delta)^2+\pi$$

$$\le\frac{1}{2}\xi^T\Big(I_M\otimes(P\tilde{A}+\tilde{A}^TP)-\frac{\delta}{4}\mathcal{L}_1^s\otimes\tilde{C}^T\tilde{C}\Big)\xi$$

$$+\frac{\delta}{2}\sum_{i=1}^{M}\sum_{j=1}^{M}((1-\tilde{\sigma}_{ij})\nu_{ij}-\bar{\sigma}_{ij})\Upsilon_{ij}$$

$$+\frac{\delta}{2}\sum_{i=1}^{M}\sum_{\epsilon=M+1}^{M+N}((1-\tilde{\sigma}_{i\epsilon})\nu_{i\epsilon}-\bar{\sigma}_{i\epsilon})\Upsilon_{i\epsilon}$$

$$-\sum_{i=1}^{M}\sum_{j=1}^{M}\frac{\iota_{ij}}{8}(\varsigma_{ij}-\delta)^2-\sum_{i=1}^{M}\frac{\iota_i}{4}(\varsigma_i-\delta)^2$$

$$+\pi+\frac{\delta}{2}\tilde{\mu}e^{-\bar{\mu}t} \tag{26}$$

where $\tilde{\mu}e^{-\bar{\mu}t}=\max\{\sum_{i=1}^{M}\sum_{j=1}^{M}\tilde{\mu}_{ij}e^{-\bar{\mu}_{ij}t}+\sum_{i=1}^{M}\sum_{\epsilon=M+1}^{M+N}\tilde{\mu}_{i\epsilon}e^{-\bar{\mu}_{i\epsilon}t}+\sum_{i=1}^{M}\sum_{\epsilon=M+1}^{M+N}(1+2\rho_i\varsigma_i)b_i^s\tilde{\mu}_\epsilon e^{-\bar{\mu}_\epsilon t}\}$.

At this point, the following two cases are considered.

1) $s\in\Xi_1$. In this case, one has

$$\dot{V} \le -c_1 V - \frac{1}{2}(1 - c_1\lambda_{\max}(P))\|\xi\|^2 - \frac{\delta}{2}\sum_{i=1}^{M}\sum_{j=1}^{M}(\bar{\sigma}_{ij} - (1-\tilde{\sigma}_{ij})\nu_{ij} - c_1)\Upsilon_{ij} - \frac{\delta}{2}\sum_{i=1}^{M}\sum_{\epsilon=M+1}^{M+N}(\bar{\sigma}_{i\epsilon} - (1-\tilde{\sigma}_{i\epsilon})\nu_{i\epsilon} - c_1)\Upsilon_{i\epsilon} - \sum_{i=1}^{M}\sum_{j=1}^{M}\frac{1}{8}(\iota_{ij} - \frac{c_1}{\kappa_{ij}})(\varsigma_{ij}-\delta)^2 - \sum_{i=1}^{M}\frac{1}{4}(\iota_i - \frac{c_1}{\kappa_i})(\varsigma_i - \delta)^2 + \pi + \frac{\delta}{2}\tilde{\mu}e^{-\bar{\mu}t} \le -c_1 V - \frac{1}{2}(1 - c_1\lambda_{\max}(P))\|\xi\|^2 + \pi + \frac{\delta}{2}\tilde{\mu}e^{-\bar{\mu}t} \tag{27}$$

where we have used the facts that $\mathcal{L}_1 > 0$, $\delta \ge 4/\bar{\lambda}_{\min}$, P is the solution of (10), $\min_{i,j\in\mathcal{V}_{\mathcal{F}}}\{\iota_{ij}\kappa_{ij}, \iota_i\kappa_i\} < 1/\lambda_{\max}(P)$, $c_1 = \min_{i,j\in\mathcal{V}_{\mathcal{F}},\epsilon\in\mathcal{V}_{\mathcal{L}}}\{\iota_{ij}\kappa_{ij}, \iota_i\kappa_i, \bar{\sigma}_{ij} - (1-\tilde{\sigma}_{ij})\nu_{ij}, \bar{\sigma}_{i\epsilon} - (1-\tilde{\sigma}_{i\epsilon})\nu_{i\epsilon}\}$.

2) $s \in \Xi_2$. In this case, one has

$$\dot{V} \le c_2 V - \frac{1}{2}(c_2\lambda_{\min}(P) - \lambda_{\max}(P\tilde{A} + \tilde{A}^T P))\|\xi\|^2 - \frac{\delta}{2}\sum_{i=1}^{M}\sum_{j=1}^{M}(\bar{\sigma}_{ij} - (1-\tilde{\sigma}_{ij})\nu_{ij} + c_2)\Upsilon_{ij} - \frac{\delta}{2}\sum_{i=1}^{M}\sum_{\epsilon=M+1}^{M+N}(\bar{\sigma}_{i\epsilon} - (1-\tilde{\sigma}_{i\epsilon})\nu_{i\epsilon} + c_2)\Upsilon_{i\epsilon} - \frac{1}{8}\sum_{i=1}^{M}\sum_{j=1}^{M}(\iota_{ij} + \frac{c_2}{\kappa_{ij}})(\varsigma_{ij}-\delta)^2 - \frac{1}{4}\sum_{i=1}^{M}(\iota_i + \frac{c_2}{\kappa_i})(\varsigma_i - \delta)^2 + \pi + \frac{\delta}{2}\tilde{\mu}e^{-\bar{\mu}t} \le c_2 V - \frac{1}{2}(c_2\lambda_{\min}(P) - \lambda_{\max}(P\tilde{A} + \tilde{A}^T P))\|\xi\|^2 + \pi + \frac{\delta}{2}\tilde{\mu}e^{-\bar{\mu}t} \tag{28}$$

where we have used the facts that $\mathcal{L}_1 \ge 0$ and $c_2 > \lambda_{\max}(P\tilde{A} + \tilde{A}^T P)/\lambda_{\min}(P)$.

For above two cases, it is not hard to find that if $\|\xi\|^2 \ge 2\pi/\tilde{\pi}$, we have $\dot{V} \le cV + (\delta/2)\tilde{\mu}e^{-\bar{\mu}t}$, where $\tilde{\pi} = \min\{1 - c_1\lambda_{\max}(P),\ c_2\lambda_{\min}(P) - \lambda_{\max}(P\tilde{A} + \tilde{A}^T P)\}$, and $c = -c_1$ if $s \in \Xi_1$ and $c = c_2$ if $s \in \Xi_2$. Furthermore, it follows from (14) that $V_{\varrho(t_{\tilde{k}})} = V_{\varrho(t_{\tilde{k}+1})}$. Thus, applying the comparison lemma gives

$$V \le e^{c(t-t_0)}V(t_0) + \frac{\delta}{2}\tilde{\mu}\int_{t_0}^{t} e^{c(t-\tau)}e^{-\bar{\mu}\tau}d\tau = e^{-c_1T^+(t_0,t) + c_2T^-(t_0,t)}V(t_0) + \frac{\delta}{2}\tilde{\mu}\int_{t_0}^{t} e^{-c_1T^+(\tau,t) + c_2T^-(\tau,t)}e^{-\bar{\mu}\tau}d\tau. \tag{29}$$

Note that (11) can be converted to

$$-c_1T^+(t_0,t) + c_2T^-(t_0,t) \le -c^*(t - t_0). \tag{30}$$

Substituting (30) into (29) gives

$$V \le e^{-c^*(t-t_0)}V(t_0) + \frac{\delta}{2}\tilde{\mu}\int_{t_0}^{t} e^{-c^*(t-\tau)}e^{-\bar{\mu}\tau}d\tau = e^{-c^*(t-t_0)}V(t_0) + \frac{\delta}{2}\tilde{\mu}\Delta(t) \tag{31}$$

where $\Delta(t)$ is defined as

$$\Delta(t) = \begin{cases} e^{-c^*t}(t - t_0), & \text{if } c^* = \bar{\mu} \\ \dfrac{1}{c^* - \bar{\mu}}(e^{-\bar{\mu}t} - e^{(c^*-\bar{\mu})t_0}e^{-c^*t}), & \text{if } c^* \ne \bar{\mu}. \end{cases} \tag{32}$$

At this point, in light of (14), it is not hard to conclude that ς_{ij} and ς_i are uniformly ultimately bounded, and

$$\lim_{t\to\infty}\|\xi\|^2 \le \frac{2\pi}{\tilde{\pi}}. \tag{33}$$

Denote $\psi_i = x_i - \bar{C}_i f_i - d_i\Pi_i\sum_{\epsilon=M+1}^{M+N}(1/N)x_\epsilon$, $i \in \mathcal{V}_{\mathcal{F}}$. Using (1), (2), (5f) and some existing knowledge gives

$$\dot{\psi}_i = (A_i + B_iE_{1i})\psi_i + B_iE_{2i}\xi_i + B_iE_{1i}(\hat{x}_i - x_i) + B_iE_{2i}d_i\sum_{\epsilon=M+1}^{M+N}\frac{1}{N}(\zeta_\epsilon - x_\epsilon) + A_i\bar{C}_if_i - \bar{C}_i\dot{f}_i + B_iz_i. \tag{34}$$

According to Step 1 and Step 2 in Algorithm 1, it is not hard to verify that

$$\lim_{t\to\infty}(A_i\bar{C}_if_i - \bar{C}_i\dot{f}_i + B_iz_i) = 0_{n_i}. \tag{35}$$

By noting that $A_i + B_iE_{1i}$ is Hurwitz, $\lim_{t\to\infty}\|\xi\|^2 \le 2\pi/\tilde{\pi}$, $\lim_{t\to\infty}\hat{x}_i = x_i$ and $\lim_{t\to\infty}\zeta_\epsilon = x_\epsilon$, it follows from (34) that:

$$\lim_{t\to\infty}\|\psi_i\|^2 \le \frac{2\pi\|\tilde{P}_iB_iE_{2i}\|^2}{\tilde{\pi}} \tag{36}$$

where $\tilde{P}_i \in \mathbb{R}^{n_i\times n_i} > 0$ is the solution of the following equation:

$$\tilde{P}_i(A_i + B_iE_{1i}) + (A_i + B_iE_{1i})^T\tilde{P}_i + 2I_{n_i} = 0_{n_i}. \tag{37}$$

It can be further obtained that

$$\lim_{t\to\infty}\|y_i(t) - f_i(t) - d_i\sum_{\epsilon=M+1}^{M+N}\frac{1}{N}y_\epsilon(t)\| \le \|C_i\|\|\tilde{P}_iB_iE_{2i}\|\sqrt{\frac{2\pi}{\tilde{\pi}}} \tag{38}$$

which indicates the MASs (1) achieve BTVOFT.

Next, we will present that no agents exhibit the Zeno behavior. Firstly, the proof of ruling out the Zeno behavior for the edge (i,j) is given. The following four cases are considered.

1) Both t_k^{ij} and t_{k+1}^{ij} are determined by $\Omega_{ij}(t) \ge 0$. Denote $\tilde{e}_{ij} = e^{\tilde{A}(t-t_k^{ij})}\zeta_i(t_k^{ij}) - \zeta_i$, $t \in [t_k^{ij}, t_{k+1}^{ij})$. Using (5c) gives

$$\dot{\tilde{e}}_{ij} = \tilde{A}\tilde{e}_{ij} + \Gamma\Big(\sum_{j=1}^{M}\varsigma_{ij}|w_{ij}^{\varrho(t)}|\hat{y}_{ij} + \varsigma_i b_i^{\varrho(t)}\sum_{\epsilon=M+1}^{M+N}\hat{y}_{i\epsilon}\Big). \tag{39}$$

According to some knowledge mentioned above, it is not hard to find that $\Gamma(\sum_{j=1}^{M}\varsigma_{ij}|w_{ij}^{\varrho(t)}|\hat{y}_{ij} + \varsigma_i b_i^{\varrho(t)}\sum_{\epsilon=M+1}^{M+N}\hat{y}_{i\epsilon})$ is bounded. It follows from (39) that:

$$\frac{d\|\tilde{e}_{ij}\|}{dt} \le \|\dot{\tilde{e}}_{ij}\| \le \|\tilde{A}\|\|\tilde{e}_{ij}\| + \Lambda_i \tag{40}$$

where Λ_i represents the upper bound of $\Gamma(\sum_{j=1}^{M}\varsigma_{ij}|w_{ij}^{\varrho(t)}|\hat{y}_{ij}+\varsigma_i b_i^{\varrho(t)}\sum_{\epsilon=M+1}^{M+N}\hat{y}_{i\epsilon})$. In light of the comparison lemma, we have

$$\|\tilde{e}_{ij}\| \leq \frac{\Lambda_i}{\|\tilde{A}\|}(e^{\|\tilde{A}\|(t-t_k^{ij})}-1) \tag{41}$$

where we have used the fact that $\tilde{e}_{ij}(t_k^{ij})=0$. It can be further obtained that

$$\|e_{ij}\| \leq \|\tilde{C}\|\|\tilde{e}_{ij}\| \leq \frac{\|\tilde{C}\|\Lambda_i}{\|\tilde{A}\|}(e^{\|\tilde{A}\|(t-t_k^{ij})}-1). \tag{42}$$

Moreover, it is not hard to verify that the event-triggered function (7a) satisfies $\Omega_{ij}(t)\leq 0$ if

$$\|e_{ij}\| \leq \sqrt{\frac{\tilde{\mu}_{ij}e^{-\bar{\mu}_{ij}t}+\nu_{ij}\Upsilon_{ij}}{(1+2\rho_{ij}\varsigma_{ij})|w_{ij}^{\varrho(t)}|}}. \tag{43}$$

Combining (42) and (43) gives

$$t_{k+1}^{ij}-t_k^{ij} \geq \frac{1}{\|\tilde{A}\|}\ln\left(1+\frac{|\tilde{A}\|}{\|\tilde{C}\|\Lambda_i}\sqrt{\frac{\tilde{\mu}_{ij}e^{-\bar{\mu}_{ij}t}+\nu_{ij}\Upsilon_{ij}}{(1+2\rho_{ij}\varsigma_{ij})|w_{ij}^{\varrho(t)}|}}\right). \tag{44}$$

Note that the right-hand-side of (44) always exists and is strictly positive for any finite time.

2) Both t_k^{ij} and t_{k+1}^{ij} are determined by $w_{ij}^{\varrho(t)}\neq w_{ij}^{\varrho(t_k^{ij})}$. Since the switching time intervals are non-vanishing, $t_{k+1}^{ij}-t_k^{ij}$ is strictly positive.

3) t_k^{ij} is determined by $w_{ij}^{\varrho(t)}\neq w_{ij}^{\varrho(t_k^{ij})}$ and t_{k+1}^{ij} is determined by $\Omega_{ij}(t)\geq 0$. Since $e_{ij}(t_k^{ij})=0$, the proof is similar to case 1).

4) t_k^{ij} is determined by $\Omega_{ij}(t)\geq 0$ and t_{k+1}^{ij} is determined by $w_{ij}^{\varrho(t)}\neq w_{ij}^{\varrho(t_k^{ij})}$. According to case 2) and case 3), one can see that even if the communication network is switched immediately after t_k^{ij} determined by $\Omega_{ij}(t)\geq 0$, the event will not be triggered immediately again. Thus, this case does not exhibit the Zeno behavior.

Combining above four cases, the Zeno behavior for the edge (i,j) is ruled out. Similarly, the Zeno behavior for the edge (i,ϵ) and leaders can also be ruled out. The details are skipped for brevity. ■